2 Vessel-Generated Waves

This chapter gives a review of the basic concepts of vessel wave generation and the pattern and characteristics of the resulting waves. For more detail see Havelock (1908), Sorensen (1966b), Sorensen (1973), and Thompson (1887).

Wave Generation

As a vessel moves across the surface of a body of water, there is flow back past the vessel hull relative to the hull. For a vessel moving in still water, the flow approaches the vessel at a relative speed that is equal and opposite to the vessel speed. At the vessel bow, the flow velocity increases relative to the vessel and is deflected from a straight path by the oblique hull surface. A pressure gradient acting along the hull is required to cause this flow acceleration. The magnitude of the pressure gradient and total pressure change depend on the vessel speed, the hull surface geometry, and the channel cross-section shape if the channel is relatively shallow and/or narrow. If the vessel is moving in a confined channel, the flow acceleration and resulting pressure gradient will be greater than if the vessel is moving across a wide and deep water body.

For a common vessel hull shape, the pressure rises in the vicinity of the bow, then falls to below the free stream pressure over the midsection of the vessel, and rises again at the stern. The water surface profile along the hull responds to this pressure distribution, causing the surface to rise at the bow and stern and to fall along the midsection. The pressure gradient and water surface rise at the stern will usually be less than at the bow owing to flow separation at the rear section of the hull. If the stern of the vessel is square rather than tapered, the flow separation will be significant, and there will be a negligible pressure rise compared with that which occurs at a tapered stern.

When responding to the sharp pressure gradients at the bow and possibly at the stern, which induce a rapid rise and fall in the water surface, inertia causes the water surface to lag behind its equilibrium position and produces a surface oscillation. This, in turn, produces the pattern of free waves that propagate out from the vessel. Consequently, the hull pressure distribution and resulting height of waves generated by the vessel depend on the relative velocity of flow past the hull, the hull geometry, and the clearance between the hull and the channel side and bottom. The period and direction of propagation of the vessel-generated free waves depend only on the vessel speed and the water depth.

The pressure rise at the bow and stern and the pressure drop along the vessel midsection also cause vessel sinkage and trim. Sinkage and trim are particularly significant in confined channels where the resulting pressure variations are greater for a given vessel hull form and speed. In a relatively confined channel, the raised water levels at the bow and stern and the lowered water level along the midsection can extend to the channel bank and affect its stability.

Thus, for increasing vessel speed or for the same speed in increasingly confined channels, the generated wave heights would increase. The increase in generated wave heights occurs provided that the vessel does not plane. For some lighter vessels at higher speeds, the hydrodynamic forces on the hull lift the vessel (planing) so it "skims" the water surface. When a vessel planes, the wave heights are lower than they otherwise would be and do not noticeably increase with increasing vessel speed.

Wave Pattern and Characteristics, Deep Water

Figure 1 shows the resulting pattern of wave crests generated by the bow of a vessel moving in deep water. (Only the waves in the vicinity of the vessel are

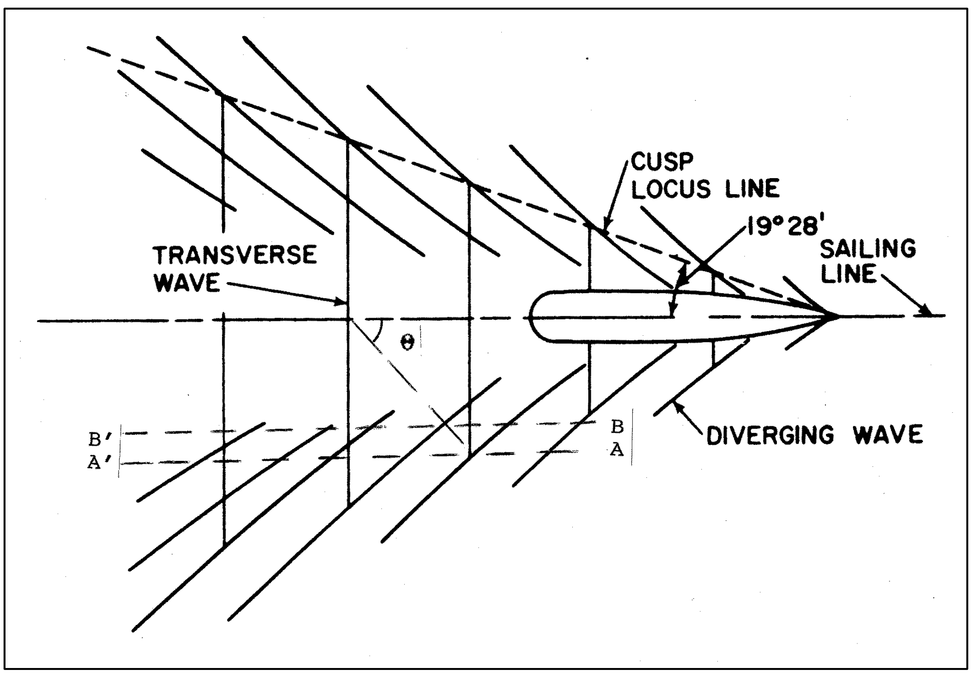

Figure 1. Deep water wave crest pattern by the bow of a moving vessel

shown. The pattern would spread out from the vessel with decreasing wave amplitudes until no longer noticeable.) The pattern consists of symmetrical sets of diverging waves that move obliquely out from the sailing line and a single set of transverse waves that move in the direction of the sailing line. The transverse and diverging waves meet to form cusps located along a pair of lines that form an angle of 19°28' with the sailing line. The highest waves in the pattern are found along this cusp locus line. If the vessel speed is increased, the lengths (and celerities) of the waves increase and the pattern spreads out but retains the same geometric shape. A similar pattern of waves, but typically with much lower amplitudes, would be generated at the vessel stern and superimposed on the pattern propagating out from the bow.

The wave pattern remains steady with respect to the vessel as the vessel travels at a speed V.[1] Thus the speed or celerity C of the waves is given by

$$C = V \cos\theta \tag{1}$$

where θ is the angle between the sailing line and the direction of wave propagation (Figure 1). For the diverging waves in deep water, the theoretical value for θ is 35°16' (Thompson 1887). (This means that the angle the crest of the diverging wave makes with the sailing line at the cusp point is 180°- 90° - 35° 16' = 54°44' Figure 1.)

For successive waves out from the vessel bow, diffraction causes the length of both the diverging and transverse wave crests to increase. This decreases the wave energy density at any point on the wave and consequently decreases the wave height. Havelock (1908) analytically demonstrated that the wave heights at the cusp points should decrease at a rate that is inversely proportional to the cube root of the distance from the bow, while the transverse wave heights at the sailing line should decrease at a rate that is inversely proportional to the square root of the distance from the bow. Consequently, the diverging waves become relatively higher than the transverse waves at increasing distances from the bow of the vessel.

Figure 2 shows the wave record measured at three distances from the sailing line for a model cargo vessel sailing in deep water. Each record shows an initial height increase to a peak at the third or fourth wave and then a gradual decrease over the remainder of the record. A comparison of Figures 1 and 2 shows that the initial wave height increase to the peak and past the peak would be for the initial diverging waves measured at successively closer points to the cusp and beyond. The remainder of the record should be dominated by the transverse waves (Figure 1). Close inspection of the wave records shows a period of about 1.19 sec for the initial waves, which are diverging waves, shifting to a period of about 1.48 sec for the remaining waves, which are predominately transverse waves. The waves that might be generated at the stern of the vessel do not appear in the record.

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[1] For convenience, symbols and abbreviations are listed in the notation (Appendix B).

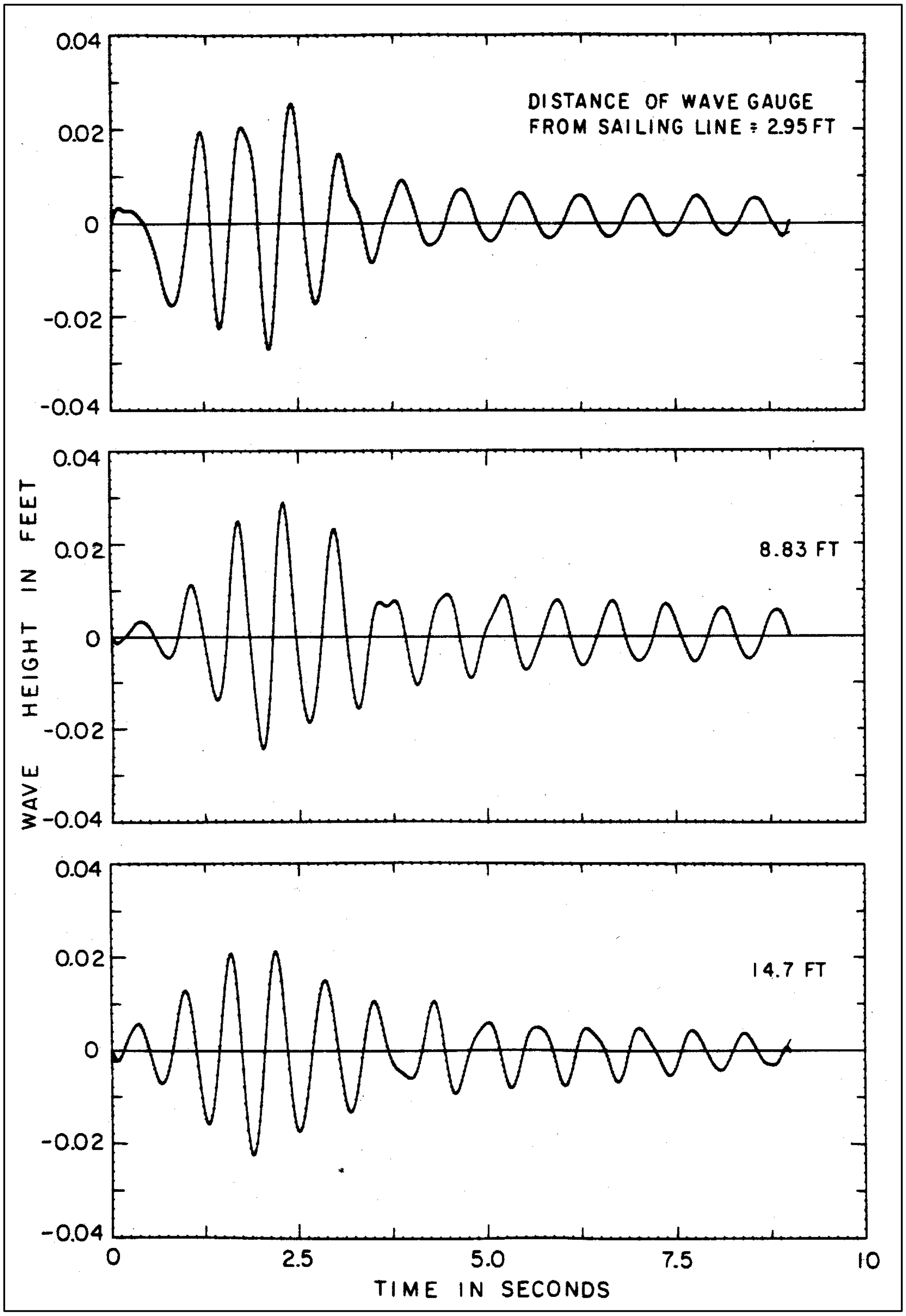

Figure 2. Wave record at three distances from the sailing line for a model cargo vessel in deep water (Das 1969)

Either they are too low to register or, more likely, the records are not sufficiently long to reach the point where they would appear.

A wave gauge located at point *A* would measure waves along line *A* - *A*' including measurement of the cusp point on the fourth diverging wave (Figure 1). A gauge located closer to the sailing line at *B* would measure waves along the line *B* - *B*' which would miss the cusp point. The records would be somewhat different with the gauge *A* record rising to more of a peak than the gauge *B* record, and the highest wave in record *A* possibly being higher than the highest wave in record *B* even though this record is measured closer to the sailing line.

Wave Pattern and Characteristics, Shallow Water

When a wave is propagating in water having a depth of less than approximately half the length of the wave, the wave-induced water particle motion reaches to the bottom and the water depth affects wave characteristics. The longer transverse waves would feel bottom first. This occurs (Sorensen 1973) when the Froude number F, defined as

$$F = \frac{V}{\sqrt{gd}} \tag{2}$$

(where d is the water depth below the still water line and g is the acceleration of gravity) reaches a value of 0.56. At a slightly lesser depth for a given vessel speed (i.e., greater F) the shorter diverging waves begin to interact with the bottom. Generally, when the Froude number is greater than 0.7 the vessel-generated wave system will begin to significantly respond to water depth effects, and noticeable changes in the wave system will occur. This condition is commonly referred to as shallow water.

As the Froude number increases from 0.7 to 1.0, several changes in the wave system will occur. Wave heights continue to rise at an increasing rate. Transverse wave heights increase at a faster rate than do diverging wave heights, so they become relatively more prominant as the Froude number approaches unity. This prominence is also greater for larger vessel drafts at a given vessel speed. The cusp locus angle increases from the deep water value of 19°28' to 90° at a Froude number of unity. This occurs because the angle that the diverging wave forms with the sailing line increases to 90°. (Consequently θ will decrease from 35°16' to zero degrees).

Figure 3 shows the water surface contours (measured by stereophotogrammetry) for one side of a model vessel moving at a speed that yields a Froude number of 0.85. Solid contour lines represent surface elevations above the still water level and dashed contour lines are below the still water level. Careful inspection shows the position of diverging and transverse waves meeting at the cusp locus line that extends obliquely back from the bow. Measurement of the

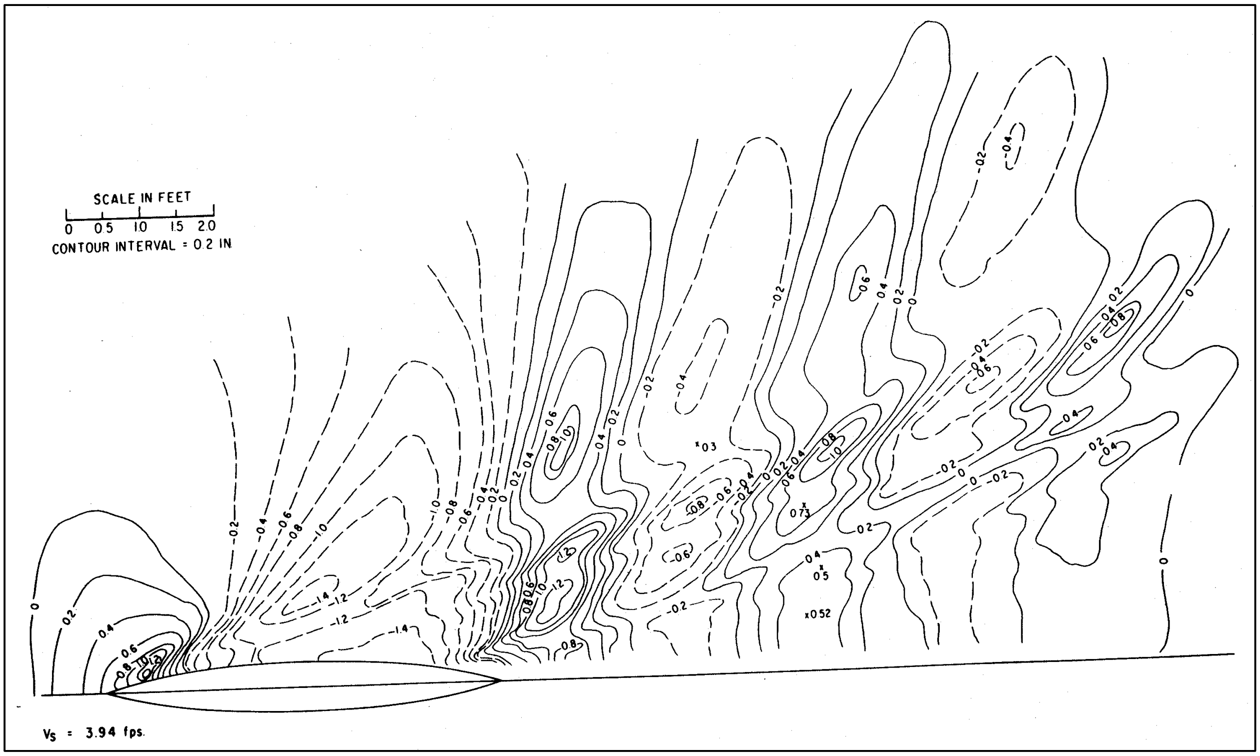

Figure 3. Measured water surface contours for the wave pattern generated by a model vessel, F = 0.85 (Sorensen 1966b)

cusp locus line angle with the sailing line yields a value of 26°. The two prominent diverging wave crests form angles of 73° and 71° respectively with the sailing line (compared to the theoretical deep water value of 54° 44').

At a Froude number of unity, when the cusp locus line angle is 90°, the transverse and diverging waves coalesce with their crests oriented perpendicular to the sailing line (at F = 1.0, $C = V = (gd)^{0.5}$). Also, as the Froude number approaches unity, the leading transverse and diverging waves are greatly accentuated at the expense of the following waves (because the wave group celerity approaches the wave phase celerity). Thus, at a Froude number of unity, essentially all of the wave energy is concentrated in the large leading wave that lies perpendicular to the sailing line near the bow, and the waves immediately behind this leading wave rapidly decrease in height.

For constricted channels and Froude numbers near unity, particularly when the wave crest axis is nearly perpendicular to the embankment, the bank will limit the extension of the wave along its axis by diffraction. Consequently, the wave height would be higher than it would be if the crest were free to extend to its natural length.

A wave propagating in water cannot have a celerity that exceeds the critical velocity (i.e., $(gd)^{0.5}$). So when the vessel's Froude number exceeds unity, transverse waves cannot exist. The resulting wave system consists of a series of straight diverging waves extending back from the bow with the leading wave forming an angle with the sailing line given by

$$a = \arcsin\left(\frac{1}{F}\right) \qquad F > 1 \tag{3}$$

For vessel speeds with $F > 1$ the wave heights are less than the peak height achieved at $F = 1$ and heights decrease as the vessel speed increases (Johnson 1958, Sorensen 1966b).

In constricted channels, self-propelled vessels that do not plane cannot attain velocities where $F > 1$. They are usually limited to a velocity defined approximately by $0.9F$ (Permanent International Association of Navigation Congresses 1987). If the vessel is moving at a velocity that yields a Froude number around 0.9, owing to the reduced clearance at the hull's wider midsection, the flow velocity at the midsection will be around the critical velocity. Increasing propeller speed will not make the vessel speed increase because the propellers cannot draw any additional flow past the vessel; therefore, the vessel speed is limited.